Using a non linear constitutive law to compare Menard PMT and PLT E-moduli

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ABSTRACT: Engineers may now be bewildered by the various values of E-moduli they can get on the same soil if they are not aware of the variability of such a modulus with the strain level that the soil sustains. Today using simple softwares as Plaxis 7 may help a lot to understand the degradation of the E value when the strain level increase.
In this paper we compare the strain levels of the soil for the pressuremeter tests, the plate loading test and the tri-axial test. Using a non linear elastic plastic constitutive law (usually known as exhibiting isotropic strain hardening) we were able to find a good agreement between the various E-moduli obtained in these different tests.

1 INTRODUCTION.

At this time, there are more and more constitutive laws to model soil behaviour from micro-strains to failure. Research workers and engineers use numerical codes at various levels:

- Level 1: for routine estimation, assuming elastic parameters;
- Level 2: for advanced calculation, assuming non-linear soil stiffness and;
- Level 3: for research, using complex constitutive laws of material behaviour.

However, the use of these codes address a major practical difficulty which is related to the choice of the characteristic values to give to the relevant parameters of the constitutive laws.

These values are usually obtained from laboratory tests where distribution of stresses and strains are homogeneous and boundary conditions are well defined. However, it is always difficult to obtain undisturbed samples; moreover, the selection of samples and their size can lead to uncertainty regarding their representativity. Consequently, in-situ tests could then become an alternative means to obtain these values. In any case it is always interesting to be able to compare the field test results with laboratory tests, and, if possible, combine both results. Unfortunately it must be pointed out that the drawback of most routine in-situ tests lies in the fact that the stress and strain distribution necessary for the identification of constitutive laws is unknown.

In this respect, it is interesting to classify in-situ tests in three categories, regarding the measurement of soil stiffness:

- Category A: includes field measurements of shear waves velocity by geophysical tests;
- Category B: includes field tests such as pressuremeter tests and plate load tests which can yield E values using a simple theory;
- Category C: includes field tests for which the soil reaction cannot be modelled by a theory.

Tests of category A permit obtaining a value of the shear modulus representing the stiffness at very small strains (G_0), that is, in the true elastic domain of ground behaviour. G_0 is a function of the slope of the straight line tangent to the (q, $d\varepsilon/\varepsilon$) curve at the origin (fig.1). It is a fundamental parameter of the ground, considered as a benchmark value, and often used to normalize the other values of G for different strains spans, under the ratio G/G_0.

The tests of category B are used to obtain in-situ stiffness characteristic values G for the stress and strain levels induced by the proposed structural loads on the tested ground. These test results are derived using the elastic theory in the following way: G is a function of the slope of either the tangent to point A or of the secant AB on the (q, $d\varepsilon/\varepsilon$) curve. It can also be a function of the slope of OB. All these values can later be used taking into account the non-linear stiffness of the ground.

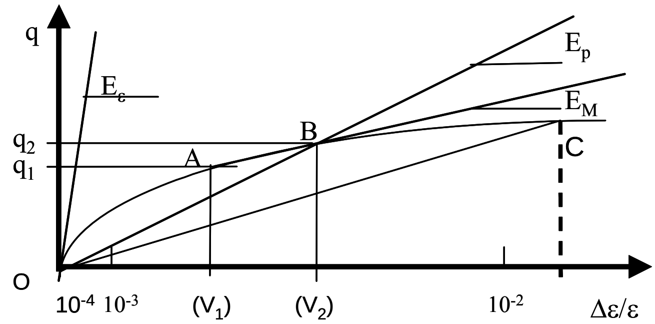


Figure 1. A typical stress-strain curve example showing tangent and secant moduli.

Tests of category C yield parameters closer to index tests; their results must be empirically correlated with the reference stiffness values obtained by the other in-situ test categories (A and B), or by laboratory tests.

This paper deals with the comparative analysis of category B field tests, here the Menard pressuremeter test (PMT) and plate load tests (PLT). Results are presented for two levels of calculations: level 1 and 2, which can be applied in engineering practice. Since it can be shown that level 1 approach leads to very different stiffness values for both types of tests, it can be of interest to move to level 2 to take into account the non-linear stiffness of soil. Further, the identification of parameters using level 2 permits to compare field results with tri-axial laboratory test results for the same level of stresses and strains.

2 PRESSUREMETER TESTS

2.1 *Level 1 – Routine calculation and analysis*

The pressuremeter originally developed by Louis Ménard in the late 50's (Ménard, 1955), is a very popular in-situ device used in various countries for most geotechnical projects, essentially for foundation design. In France, it has been standardized since the end of 60's (MEL, 1971), and in the USA since the mid 80's. The present standards are respectively AFNOR NF P 94-110-1 and ASTM D 4719-00. An ISO standard No. 22476-4 is under preparation on the same topic. The analysis of test results for routine calculations (level 1), leads to the Menard modulus E_M, among other stress-strain parameters. This modulus is determined in the pseudo-elastic zone of the curve relating the uniformly distributed pressure applied on the borehole wall to the volume change of the cavity. The theory of Lamé for thick cylinders is used assuming a linear-elastic, homogeneous and isotropic behaviour of the tested soil :

$$E_M = 2(1+\upsilon)V_0 \frac{dp}{dV} \tag{1}$$

where:

V_0 is volume of the probe measuring cell;

dp/dV is the slope of the secant AB on the pressuremeter curve in the pseudo-elastic zone;

ν is the Poisson's ratio.

The Menard modulus is a Young's modulus calculated for a Poisson's ratio of 0,33. Figure 2 presents a criterion to define the pseudo-elastic zone (Briaud, 1992). In Figure 3 another representation is proposed, which gives directly the tangent modulus G (Gambin and Jezequel, 1998).

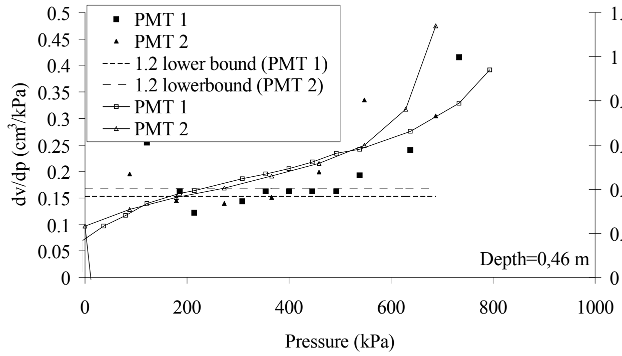


Figure 2. Definition of the pseudo-elastic zone for tests PMT1 and PMT2.

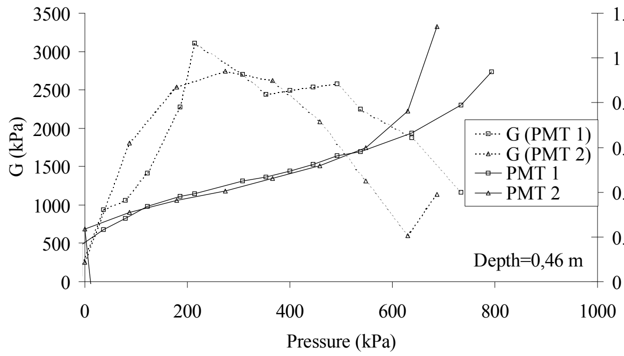


Figure 3. Proposal for obtaining tangent shear modulus G in the pseudo-elastic zone.

It must be pointed out that this modulus is related with the average stiffness exhibited by the ground associated with a determined strain level. Consequently the use of this value must be corrected by an empirical calibration factor to be applied in design, as for instance this is done in the French Code for foundation design (MEL, 1993, Clarke, 1995). Consequently Menard modulus must be considered as a test-specific design parameter.

For an unload-reload cycle a secant modulus can also be defined, which in our tests is around 2,2 times Ménard's modulus. If the cycle is selected in a cavity strain range of 0.1 – 0.2 %, the secant modulus can be used to predict settlements (Clarke, 1995).

2.2 *Level 2 - Advanced Analysis*

Since the pressuremeter test is an "in-situ" test where boundary conditions are known, the analysis of a test results has a sound theoretical basis. The procedure to identify parameters is essentially based on the nature of a "back analysis" requiring an appropriate modelling of the constitutive laws of the ground. Since the test analysis is an inverse problem, identification of parameters, analytically or numerically, depends entirely on the modelling technique (Gioda, 1985). Therefore, realistic model assumptions must be addressed, otherwise physical meaning of parameters can be loosed.

The advance analysis of test results is dealing with a non linear modelling analysis of soil behaviour. In this context three aspects must be considered:

- Elastic behaviour of ground for the domain of very small strains;
- Elasto-plastic behaviour for intermediate level of strains and;
- Non linear geometry for large strains.

Various constitutive laws were use in modelling the pressuremeter results: Cambou et Boubanga (1989), Shahrour et al. (1995), Biarez et al. (1998). In this paper we adopt a simple model developed by PLAXIS and called HSM (hardening soil model) which assumes a non linear elastic response of the soil during loading and a isotropic hardening during unloading. The geotechnical parameters which are necessary to carry out the study are well known by professionals. The main features of this model are:

- Stress dependent stiffness.
- Plastic straining due to both primary deviatoric and compression loading.
- Elastic unloading and reloading.
- Failure according to the Mohr Coulomb model.

The hyperbolic response due to tri-axial deviatoric loading, as well as the oedometer simulation, are shown in Figure 3, where the meaning of the stiffness parameters of the model is submitted.

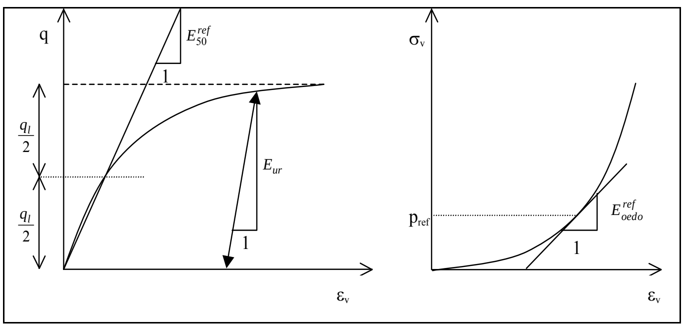


Figure 4. Schematic response for tri-axial and oedometer tests by Plaxis-HSM.

Furthermore, the model has parameters allowing changing strength and stiffness with depth, but this was not adopted in the calculations.

The identification of the parameters of this model has been done by fitting pressuremeter test data with the numerical results. However, a number of combinations of these parameters will produce the same sort of best fit, therefore it was necessary to exhibit some engineering judgement to ensure that parameters obtained are compatible with one another. Furthermore, laboratory results of the test material were also used, as the friction angle of tri-axial tests (Fleureau et al., 2002). This method of identification of parameters, known as computer-aided-modelling (CAM) is now seen as an alternative and a promising method in many geotechnical applications.

Figure 5 shows the best fit obtained for two pressuremeter test results (PMT1 and PMT2) obtained at the same depth in a embankment built with a silty sand; this soil is a residual soil from granite.

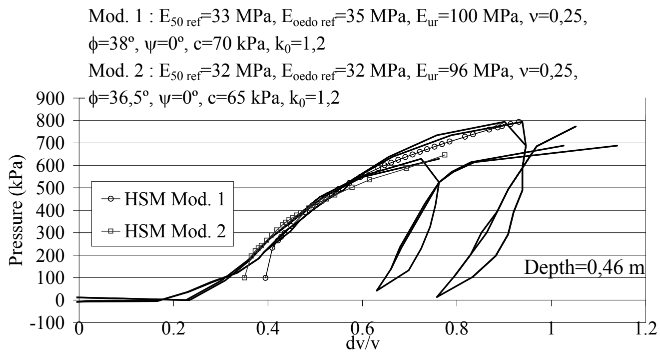


Figure 5. Best fitting curves of two pressuremeter tests using Plaxis-HSM.

3 THE PLATE LOAD TESTS

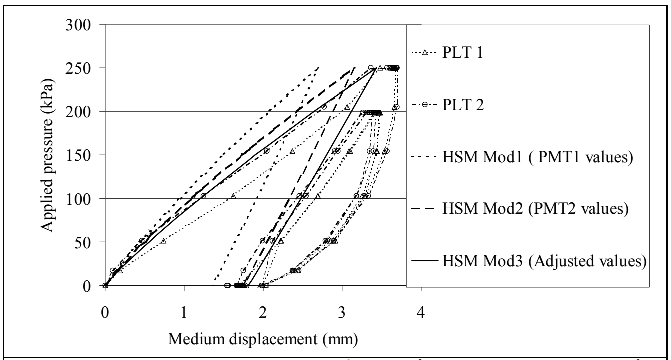
3.1 *Level 1 – Routine calculation and analysis*

The load plate test is also used in routine calculations of foundations (ASTM D1196), roads and airports (ASTM D1195, LCPC-CT2). Generally plates of large diameter are used with smaller plates on top to ensure a sufficient rigidity to obtain uniform settlements under the plate. As in the pressuremeter routine analysis, the modulus derived from the plate load test is valid only for a linear-elastic, homogeneous and isotropic ground. In this case the Boussinesq equation can be used to obtain the modulus for the first loading:

$$E = \frac{1{,}5\,Q_{appl}\;D/2\left(1-\upsilon^2\right)}{\delta} \qquad (2)$$

for the case of a rigid plate, as is the case of that used in our tests.

Figure 6 shows the results of plate load test carried out according to the LCPC procedure (LCPC-CT 2). When unload-reload cycles were realized, it is also possible, like in the pressuremeter, to define a secant modulus, which in our tests is around 1,8 times the modulus in the first loading. Figure 7 shows the increase of the secant modulus of reloading with the stress level. This is a typical behaviour of compacted unbound granular materials, where the strains under this level of stress are practically elastic (Gomes Correia and Biarez, 1999). In this domain, an increase of modulus with the increase of stress in the direction of the load is observed.



HSM Mod3: γ=18 kN/m^3, E_{50}^{ref} =27,5 MPa, E_{oedo}^{ref} =27,5 MPa, E_{ur}=82,5 MPa, m=0,5, c=70 kPa, ϕ=38º, ψ=0º, ν=0,25, k_0=1,2

Figure 6. Plate load test results according to LCPC-CT2 procedure and best fitting curves.

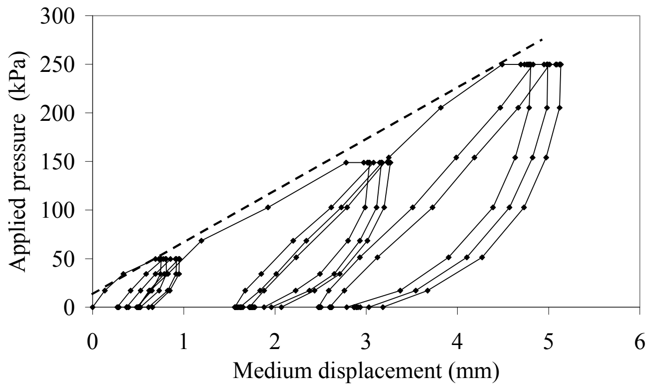


Figure 7. Plate load test results according to ASTM-D1196 standard.

3.2 *Level 2 - Advanced Analysis*

Since the plate load test gives, like pressuremeter tests, a load-settlement curve, it might be expected that it would be possible to use an inverse technique to identify parameters of a non-linear model for the ground. So, the same numerical tools used for the pressuremeter analysis were applied: Plaxis with HSM constitutive law. In a first step, the same parameters obtained before were used (fig. 6), i.e. HSM Mod.1 and HSM Mod.2. One observes a ground response with a stiffer behaviour in loading as well in unloading. This can be a consequence of a less good adaptation of the model to simulate the field of stresses and strains of plate load test, which is different from that of the pressuremeter test. Indeed, plate load test procedure induces strain levels in the ground much lower than those used in the initial fitting of the pressuremeter curve. Another explanation can be the physical nature of the real behaviour of the material. In spite of the depth of 0,46 m, pressuremeter test results can be considered as representative of the plate load test with a 0,61 m plate diameter performed on surface. It is very likely in a granular material, compacted with a vibrating roller, that the state of density close to the surface is less dense, contributing to a global less stiff response of the ground under the plate load test. One better curve fitting to the plate load test (HSM Mod.3) is obtained reducing original modulus of pressuremeter simulations by about 15 % (fig. 6).

With these model parameters from the plate load test a numerical simulation was performed in order to observe the strain distribution in depth in relation to the total surface settlement for a given load. These results led to an almost unique relationship between the ratio of the settlement at a given depth and the total settlement, and the ratio between this depth and the diameter of the plate (fig. 8). This confirms that practically (about 90 %) settlement arises from strains up to a depth about twice the diameter of the loading surface.

It is also interesting to observe the results of numerical simulations of the variation of the secant modulus with depth on the axis of the load (fig. 9). These results can also be represented by replacing depth by the corresponding strain in the ground (fig. 10), which explains the increase of the modulus with depth, as a result of a decrease of the strain level. These results put in evidence the difficulty of identification of the parameters of a non linear law from a load-settlement curve of a plate load test without having other information. A possible solution to this is the measurement of deformations in depth under the plate with strain gauges, as it was proposed by Burland (1989).

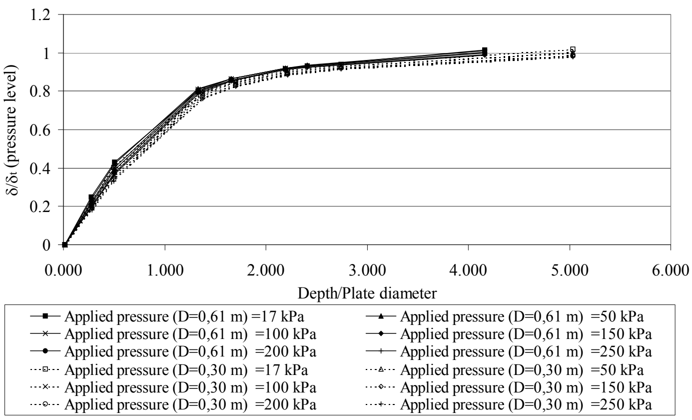


Figure 8: Ratio of the settlement at a given depth and the total settlement (δ/δ_t) as a function of the ratio between this depth and the diameter of the plate (z/D).

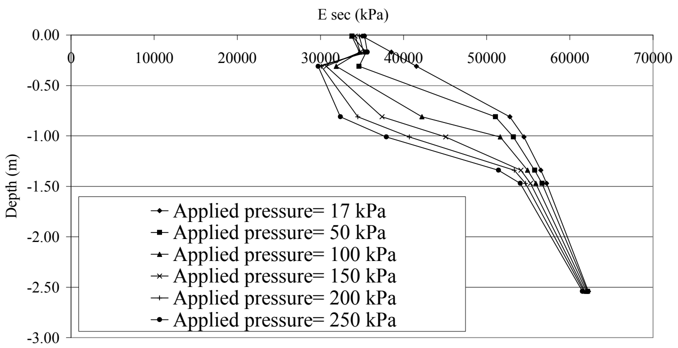


Figure 9. Variation of the secant modulus with the depth.

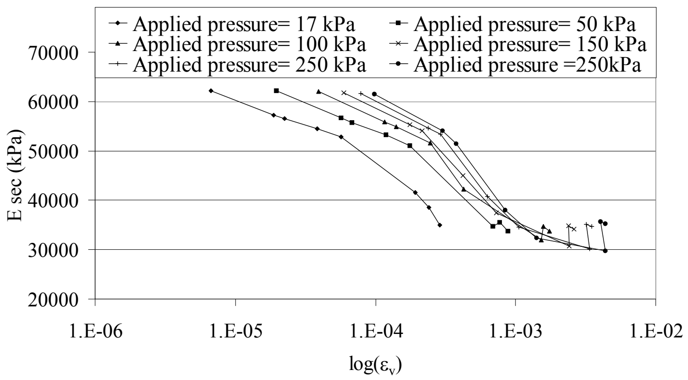


Figure 10. Variation of the secant modulus with strain.

4 COMPARISON BETWEEN MODULI FROM PRESSUREMETERS, PLATE LOAD TESTS AND TRI-AXIAL TESTS

The routine interpretations of PMT and PLT led to very different values of modulus. Besides, the Menard modulus is a tangent modulus, in the sense that it is obtained by the slope of the pseudo-elastic zone of the pressuremeter curve, while the modulus obtained by the plate load test is generally a secant modulus. In addition it is obvious that these modulus will be modified if test procedure or interpretation is modified. This is a consequence of the non-linear material behaviour, where the modulus depends on the level of stress and strain, among others. The main point is to know in practice how to use correctly these values. In fact, modelling geotechnical structures is being more and more popular, and consequently the results of category B tests must be more and more used for the identification of the model parameters. It is evident that the correctness of this identification is a function of the model adopted. So, the appropriateness of the model must be carefully analysed and confirmed.

The model chosen in this study is a compromise between a simple constitutive law of ground behaviour with the description of its more important aspects in terms of stress, strain and strength and the use of parameters having a physical meaning fully understood by practitioner engineers.

With this model, and by using the parameters deducted from pressuremeter and plate load tests, a numerical simulation of tri-axial tests is submitted on Figure 11. These simulations were done for different stress paths corresponding to those induced by the plate load test at various depths (0,1D; 0,5D; 1D and 2D). Figure 11 represents these results which express the secant and tangent modulus as a function of strain level. It can be observed that these results are very sensitive to the influence of stress paths and consequently to the depth.

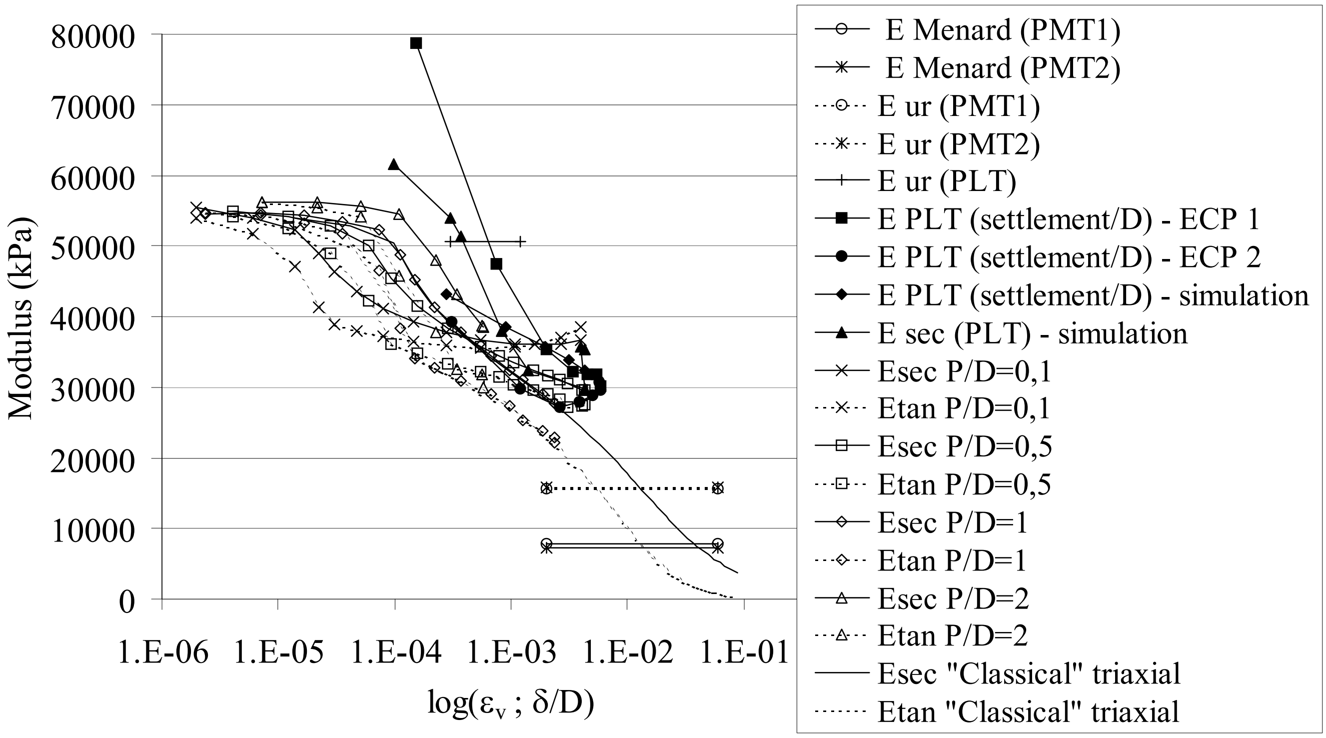


Figure 11. Moduli as a function of strain level for various numerical simulations and test analysis.

From Figure 11 it can also be noticed that the Menard modulus obtained in the routine analysis is associated to a strain level near 1 %, while the secant modulus of unload-reload of the plate load test is rather close to 0,1 %. This difference of strain levels leads to an unload-reload modulus of plate load test about three times the unload-reload modulus of pressuremeter, associated to a different strain level. On this Figure 11 the secant modulus of the plate load test (from the routine analysis) is submitted as a function of the ratio of settlement and plate diameter (tests PLT1 e PLT2). One of test results (PLT2) leads to a variation trend similar to the results of the simulation of the two tri-axial tests for the deeper stress paths. It is also interesting to observe that the ASTM D1195 procedure of plate load tests leads to levels of strains during loading variable between 0,01 % and 0,6 % (this last one for a stress of 250 kPa). Another interesting result comparable to the previous one is the variation of the secant modulus with the level of strain calculated in depth (result presented in Figure 10) for an average stress of 250 kPa applied by the plate.

5 COMPARISON BETWEEN RESULTS OF STRESS-STRAIN BEHAVIOUR FROM TRI-AXIAL AND PLATE LOAD TESTS

Hillier and Woods (2001) showed that for a non-linear elastic behaviour of the ground, the curve load-settlement of a plate load test:

$$Q = K\left(\frac{\delta}{D}\right)^n \tag{3}$$

follows the same power law as the constitutive model of the ground:

$$Q = K'\varepsilon^n \tag{4}$$

To validate this findings using our results, tri-axial test curves (vertical stress-vertical strain for different stress paths) were compared with similar plate load test results (applied vertical stress of PLT versus ratio of settlement to diameter δ/D of the plate) (fig. 12).

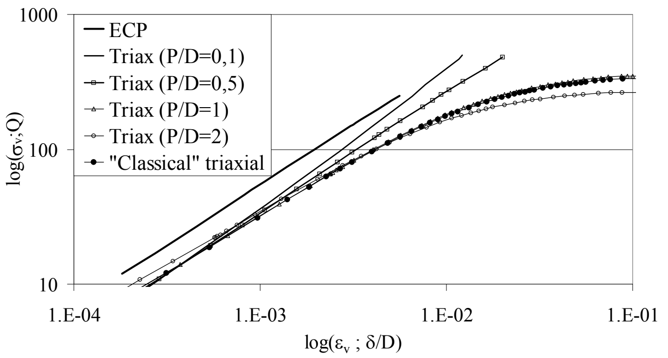


Figure 12. Comparison between tri-axial test strain and PLT strain

The analysis of these results shows a certain influence of the stress path. For the conventional stress path there is a domain of load where strains are 0,5 times the "relative strains" δ/D of the plate load test, whereas Hillier and Woods (2001) found, for a power of n=0,55, a value of 0,3. After a certain level of load it is evident that, due to the plasticity of the ground, this ratio is not valid any more.

6 CONCLUSION

Routine analyses of Menard PMT and PLT lead to E-moduli values which are quite different. Further these values would be more or less different if the way to carry out the tests is changed or the method of E values derivation is modified. In a compacted silty sand the first loading E-modulus for a PLT is 2.7 times that one of the Menard E modulus. Further the unload-reload modulus of the PLT is 3 times the unload-reloading E-modulus of PMT. This exemplifies the warning that these E-moduli values must only be used within specific rules of design.

It was shown that these apparent discrepancies between the E values stem from the fact that moduli are a function of the strain level. It is important to analyze these tests with relevant constitutive laws.

Using the previous modeling technique where the friction angle is derived from tri-axial tests, we can draw the following conclusions:

-the strain level associated with the Menard E modulus is of the order of 1 %, when the strain level associated with the PLT unload-reload modulus is about 0.1 %,

-the ASTM practice for the PLT leads to a strain level of 0.01 to 0.6 % during loading,

-these strain levels are in good agreement with the E-moduli values obtained for both types of tests,

-there is almost a single relationship between the ratio of the elementary settlement at a given depth over the total settlement and the ratio between this depth over the plate diameter. The full settlement arises within a depth equal to twice the plate diameter,

-in the non linear elastic behaviour domain of the soil, the curve which expresses the variation of the applied pressure during a PLT versus the ratio settlement over diameter is close to the curve which shows the function of the vertical stress versus the vertical strain in a tri-axial test. In the numerical modeling with a power law equal to 0.5, the ratio between the relative plate deformations or "relative strains" δ/D and the tri-axial strains was about 0.5.

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