Homogenising MPM test curves by using a hyperbolic model.

J.-P. Baud
Eurogéo, Avrainville, France

M. Gambin
Apagéo, Magny les Hameaux, France

ABSTRACT: Development of the interpretation of Ménard pressuremeter (MPM) data using the double hyperbolic model is still progressing. Simultaneously research work is undertaken to improve the starting phase of a test by a drastic decrease of both the stress relief and the remoulding around the borehole wall. Further a new software by the name of Xpressio® was developed which helps obtain MPM parameters in an industrial fashion. These advances now permit to submit MPM plots where the soil response can be modelled by a single hyperbola, a typical feature in the elastic plastic behaviour of an undisturbed soil. Then it becomes possible to characterise a homogeneous geological formation by a single hyperbolic response to cylindrical expansion, after compilation of a large number of hyperbolically modelled MPM tests curves at a job site. This powerful and versatile tool so obtained can be very helpful to the practising geotechnical engineer.

1 INTRODUCTION

The constitutive law of soils is one of the main subjects of research work for soil mechanics scientists. The stress-strain relationship obtained during the expansion of a cylindrical cavity in soils (ASTM, 2007) was analysed by Louis Ménard (1957) to describe the pseudo-elastic phase and the concept of the limit pressure in the large strains domain. Several authors confirmed that the expansion of a pressuremeter in the soil shall exhibit this limit pressure originally demonstrated theoretically by Bishop et al. (1945), among them: Gibson & Anderson (1961), Baguelin & al (1978), etc.

In the late 60's too, after the original work from Kondner (1963), Duncan & Chang (1970) proposed an hyperbolic best fit for the (e,q) graphs obtained in the laboratory on non remoulded soil samples:

$$q = e / (a + b . e)$$

Very early a simple hyperbolic fit for the final part of a MPM test plot was used to extrapolate the readings and obtain the conventional limit pressure p_{LM}, by working on the reciprocal of the volume readings. The plot of the last 3 readings in a test after the "creep pressure" on a (p, 1/V) graph is very close to a straight line. This makes it easy to derive the conventional limit pressure p_{LM} either by plotting the data on that sort of graph (Van Vambeke & D'Hemricourt, 1975), or by a simple regression calculation over n points (n = 3 and each selected p reading = pf) (TLM, 1980):

$$p_{LM} = a/(V_p + V_1) + b \qquad (1)$$

where

– V_p is the volume of the probe when deflated
– V_1 is the probe additional volume to reach the initial volume of the pseudo elastic range used to calculate the conventional E_M modulus
– and the coefficients a and b are given below:

$$a = \frac{\sum p/V - (\sum 1/V \cdot \sum p)/n}{\sum 1/V^2 - (\sum 1/V)^2/n} \qquad b = \frac{\sum p - a.(\sum 1/V)}{n}$$

The same way, the best fit of a full MPM test plot can be obtained by adding a short straight line between two hyperbolas arcs with opposite concavities. This method which involves the calculation of 6 mathematical parameters has proven to be perfectly suitable to model the behaviour of any type of soil subject to an expansion test in a borehole (Van Vambeke & D'Hemricourt, 1978).

The application of the double hyperbola method to any set of MPM data was exemplified earlier (Baud & al., 1992), constituting a back analysis method for modelling the expansion of a cylindrical cavity in the ground submitted to a uniformly increasing pressure. The attractiveness of the method has been increased by a) the development of more refined techniques of drilling to reduce the remoulding of the borehole wall and keep stress relief to a minimum (Arsonnet & al., 2005), resulting in minimising the first hyperbola arc and the straight line in the modelling, b) the use of a new industrial software, by the name of Xpressio®, to obtain the MPM parameters from automatically logged readings (Apagéo, 2006).

2 ANALYSIS OF THE DOUBLE HYPERBOLIC BEST FIT

2.1 *Statistical treatment of MPM curves*

The refined analysis of the expansion of such a cavity during one single test is certainly useful for giving to a soil mechanics scientist the opportunity to obtain a stress-strain law which is valid for all levels of strains ranging from rest condition up to failure for a soil at one particular depth.

For the geotechnical engineer, the statistical analysis of a large number of tests in a lithologically homogeneous formation investigated by several boreholes drilled throughout its whole thickness is more important. It is one of the fundamental aspects of the pressuremeter method, for which Louis Ménard proposed to take into consideration the reciprocal or harmonic mean of the E_M moduli and the geometrical mean for the p_{LM} limit pressures. Whenever possible, histograms of E_M and p_{LM} for each geological formation are carefully analysed and compared with the lognormal distribution (Cassan, 1978). The E_M/p_{LM} ratio, which, to a greater or lesser extent, reflects the stretched appearance of the MPM second hyperbola, is also a criterion to discriminate between families of tests.

2.2 *The help that the hyperbolic modelling can provide*

Can the next step be a comparison of the modelled curves by superposition? In the case of a site investigation involving several heterogeneous soils, this ambition would soon result in a series of conflicts, not only due to the natural diversity of the limit pressures but also to the shape of the curves (expressed by their E_M/p_{LM} value), and the contingencies particular to each type of curve (variable probe volume at borehole wall contact and variable curvature near the contact pressure) related to the quality of the preliminary drilling.

The test curves to be compared with each other must therefore be sorted first into categories representing a relevant set from a geotechnical standpoint, or in other words which represent a type of soil and lithology exhibiting a relatively similar behaviour. It is not necessary to say that this selection cannot be automated, it constitutes part of the geotechnical engineer expertise, based firstly on the description of the cuttings during drilling and, if possible, from borehole logs taken from core sample examination at the same site. The statistical analysis referred to above should assist with making this choice.

The statistical selection can be largely helped by looking for relatively tight cluster of dots in the [log(p^*_{LM}), log(E_M/p^*_{LM})] MPM tests Pressiorama® spectral diagram (Baud, 2005), where $p^*_{LM} = p_{LM} - s_{hs}$, with s_{hs} being the soil horizontal pressure at rest.

Then a synoptical presentation to compare the various curves within one of these cluster can be conducted by assigning common axes to curves in p* and e, where

$p^* = p - s_{hs}$ and

$e = (V - V_1)/V_p$ where

p is the corrected pressure reading

V the corrected volume reading

V_1 the start volume for the pseudo-elastic phase,

V_p the volume of the probe when the contact with the borehole wall is reached.

The e value varies between 0 and 1.

This change can be done easily by considering the 4 coefficients a1, …a4 which appear in the tags of the final diagram for each MPM test computed with Xpressio® program as single hyperbolic fitting:

$$v = a_1 + a_2.p + a_3 / (a_4 - p) \qquad (2)$$

This equation describes the soil reaction when the start phase of MPM test to the original at rest condition is eliminated when neglecting the first readings.

2.3 *Some examples*

The few examples of tests selection provided below come from various job sites, where the tests were carried out

- either within a pre-drilled hole, as at Limoges, France, in weak rock (Figure 1)

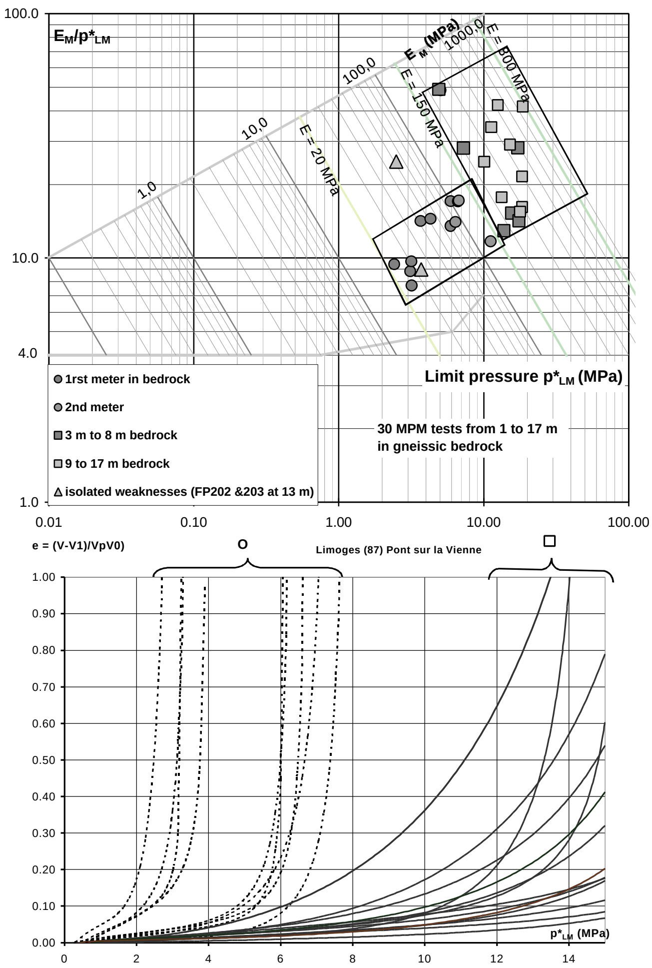

Figure 1 – Pressuremeter tests in weathered gneiss

- or in a cavity created using an open end slotted tube fitted with a disintegrating tool inside permitting removal of the cuttings by mud circulation (“STDTM” according to the English acronym in the next EN ISO Standard for MPM testing), here more exactly with the STAF® technique (Arsonnet & al., 2005) allowing test inside selfbored slotted tube at St-Nom-la-Bretèche, and Villeneuve-le-Roi, France (Figures 2-3).

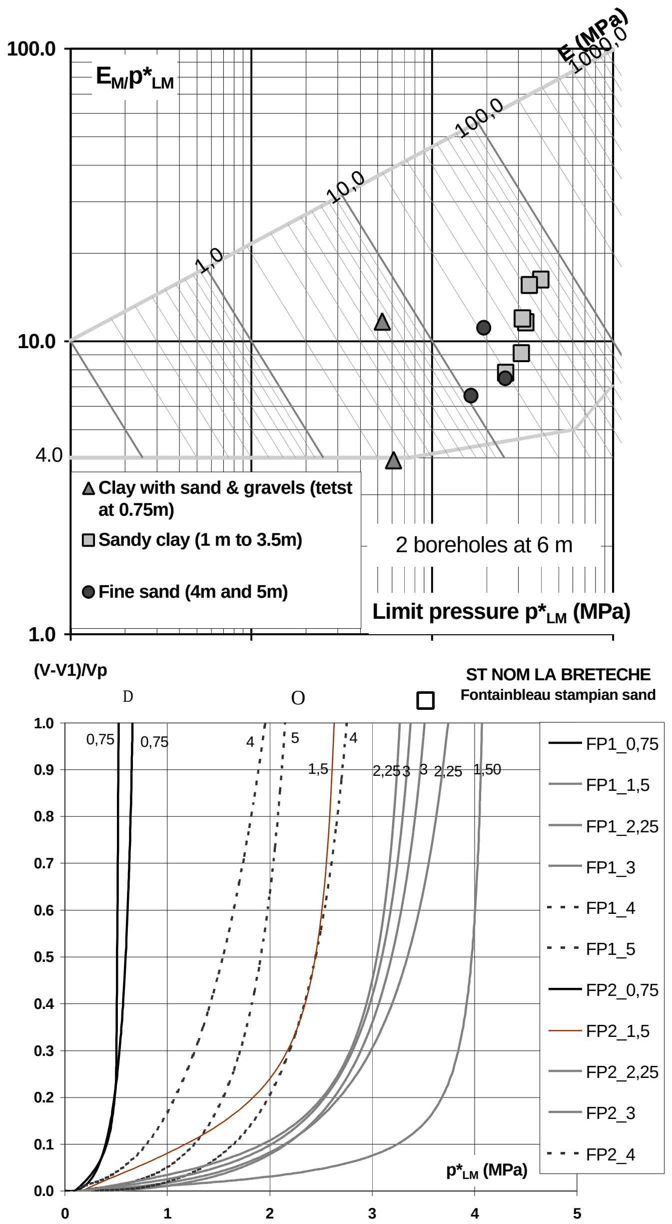

Figure 2 Pressuremeter tests in sand

In the latter case, curves comparison is facilitated by the better quality of the curves, which only exhibit a single concavity, V_1 corresponding to the first reading during the test and the first plot on the curve. The (p,e) graphs for the selected MPM tests are shown together with the Pressiorama® diagrams from where they were extracted, the tests belonging to the same cluster. Therefore the differences in the features of the MPM tests curves within the same formation, if any, are obvious. The tests results differ both by the slope of the curve at the origin which is proportional to the E_M value and by the p^*_{LM} value of the asymptotic end of the curves.

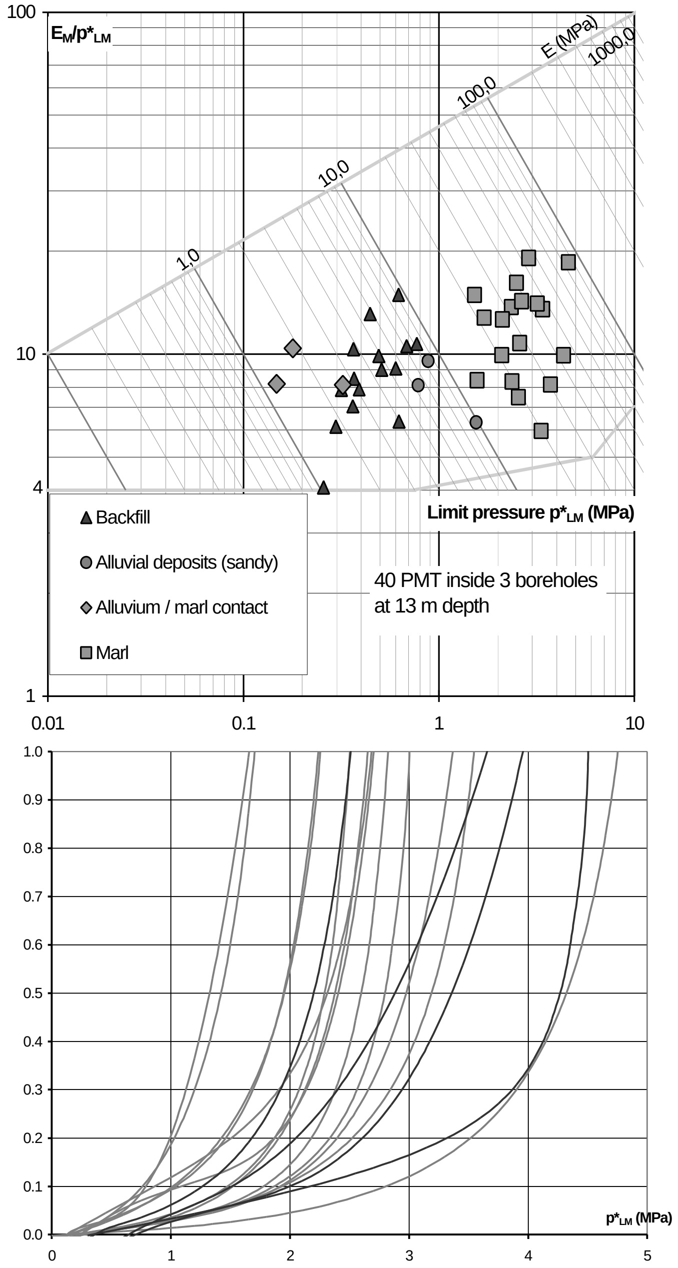

Figure 3 : Pressuremeter tests in marl.

2.4 *A single curve to characterise the soil tested at several spots*

To get a better approach to the behaviour of a given geological formation, it is possible to plot all its MPM tests curves obtained on a job site in a single reference graph (s, e), where:

s is equal to p/p_{LM}

e is equal to $(V - V_1)/V_p$.

The curves fit in the same quadrangle and a simpler hyperbolic modelling is then possible with an equation involving only 4 parameters. On each of the 3 graphs shown as examples (Figure 4) is given the mean constitutive law of the modelled curves.

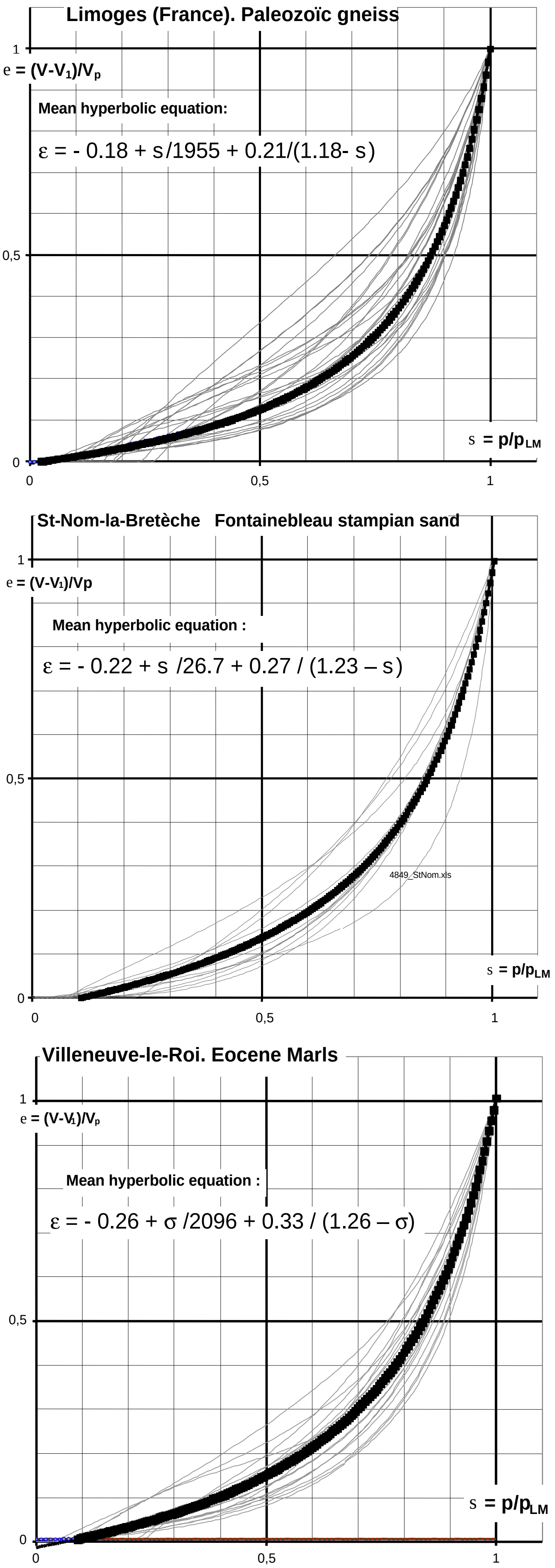

Figure 4 : (σ,ε) graphs based on Figures 1, 2 & 3 MPM results.

3 GENERALISATION OF THE HYPERBOLIC MODEL

In the double hyperbolic model, the existence of the first hyperbola arc is due to the stress relief of the soil between drilling and testing phase and to borehole wall remoulding. In the examples above, we managed to reduce and partially eliminate these disturbances. Still, in pre-drilled tests, even particularly careful ones, there is an inflection point pE and elimination of the first hyperbola is only possible by accepting a moderate adjustment of the original readings p1 and pE readings. Conversely, tests carried out by the “STDTM” technique or by the STAF® self-bored slotted tube, yield plots with a single concavity, which can be modelled by a single hyperbola arc.

3.1 *Expression of the constitutive law from the hyperbolic curve*

By partly adopting a notation that has already been proposed (Van Wambeke & D’Hemricourt, 1978), the general equation for a MPM test curve without stress relief or remoulding of the soil is expressed by the following relation between the volumetric strain e and the pressure p applied by the probe on the soil:

$$e = e_0 + \frac{p}{E_0} + \frac{R \cdot p_{LM}}{K \cdot p_{LM} - p} \qquad (3)$$

or, as for any in situ soil testing, in involving the net pressure p - p_0, where p_0 is the initial pressure at rest in the ground, equal to s_{hs}:

$$e = e_0 + \frac{p^*}{E_0} + \frac{R \cdot p^*_{LM}}{K \cdot p^*_{LM} - p^*} \qquad (4)$$

This equation is very similar to those found out by other methods to describe soil behaviour by hyperbolic models. The 4 parameters, which correspond to the addition of a straight-line segment and a hyperbola arc, are:

– e_0 the ordinate at the origin of the deformation, a quantity that is negative.

– E_0 the inverse of the slope of the curve; this quantity has the dimension of a modulus, and defines the oblique asymptote with the preceding quantity (according to the equation $e = e_0 + p/E_0$).

– R a dimensionless coefficient function of the minimum radius of curvature of the hyperbola.

– K the ratio between the “conventional” limit pressure p^*_{LM} and the “true” limit pressure p^*_{L8} which is the abscissa of the second asymptote, a vertical one.

The apparent complexity of the double-hyperbolic method is therefore relatively simplified since these 4 parameters only correspond to 3 degrees of freedom for the curve, because they are interlinked in the following manner:

– $K = p^*_{L8} / p^*_{LM}$ is set by definition and

– the curvature (or failure) coefficient R, given by:

$$R = K\ (K - 1)\ \left(1 - \frac{p^*_{LM}}{E_0}\right)$$

is determined by K, E_0, p_0 and p_{LM}

– and the ordinate at the origin is fully determined by the two previous parameters:

$$e_0 = -\frac{R}{K}$$

The graph for the MPM test curve which results from this modelling is very simple and fulfils the law of the elastic-plastic behaviour as expected by any practitioner working in the field of MPM testing (Figure 5).

This expression of the soil reaction under a radial expansion can be proposed as a general law of behaviour for a material. It provides a comprehensive model from small strains up to failure. It can be used in existing calculation codes after setting a limited number or parameters (p_0, p_{LM}, E_0, K), and can be experimentally adjusted from a MPM site investigation adapted to the complexity of the construction project:

$$e = -\frac{R}{K} + \frac{s}{E_0} + \frac{R \cdot p^*_{LM}}{K \cdot p^*_{LM} - s} \qquad (5)$$

3.2 *Derived parameters*

The knowledge of this comprehensive constitutive law permits to obtain numerical values for the parameters that may be deduced from a MPM test. We shall only give the example of the E-moduli, since the shear strength cannot theoretically be derived from the MPM tests results (Ménard, 1963).

3.2.1 *The various types of E-modulus.*

By derivation of the curve on the e, s graph, and adopting a Poisson's ratio of ? = 1/3, as Ménard suggested, several E-moduli can be obtain (Figure 5):

- Firstly the secant modulus E_{Ms} by drawing a secant from the origin of the MPM curve:

$$E_{Ms} = \frac{8}{3} \cdot (1+e/2)\frac{(s - s_0)}{e} \qquad (6)$$

The E_{Ms} function can be plotted from the MPM curve graph as a function of the second intersection point abscissa on the MPM curve. This curve exhibits a very small convexity, with an upward concavity (Figure 7). For s_{LM} - s_0 = 0.5 we obtain the Ménard modulus E_M which is between its initial value (E_{Mti}) and its final value which is such that E_M / (? s_{LM} - ? s_0) = 4. This constant limit value correspond to that one for a remoulded material exhibiting the highest void index (Baud, 2005)

- And secondly the tangent modulus E_{Mt} :

$$E_{Mt} = \frac{8}{3} \cdot \frac{(1+e)(K.p^*_{LM} - s)^2}{R.p^*_{LM} + (K.p^*_{LM} - s)^2/E_0} \qquad (7)$$

The E_{Mt} function can be plotted too as a function of the point of contact on the MPM curve. It decreases, starting from the value it has when it is the initial tangent modulus E_{Mti} valid for micro-deformations, or E_0, and tends towards zero for s = $K.p_{LM}$.

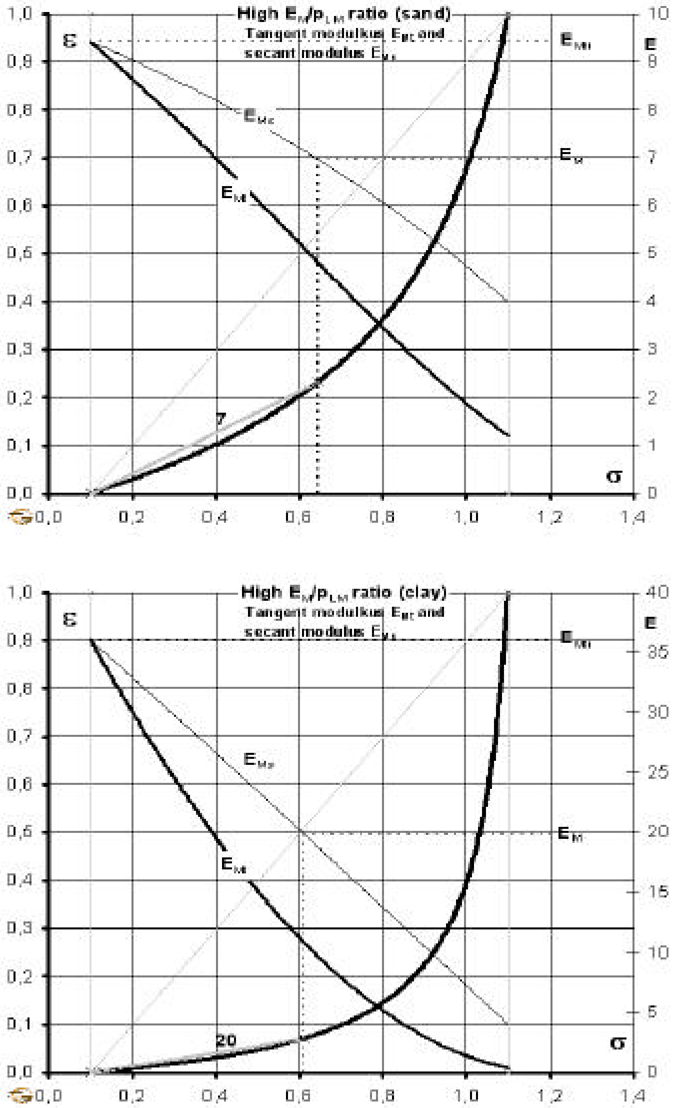

Figure 5: Example of the secant and tangent E-moduli E_{Ms} & E_{Mt} versus the stress level s (typical hyperbolic MPM curves in sandy or clayey soils).

3.2.2 *The G/Go law from the MPM tests*

Soil Mechanics research workers are more and more interested in very small strains. Although the MPM works in the (10-2, 10-1) strain range, the extrapolation of the MPM curves toward these small strains using the hyperbolic law can be tested, since it will yield a G0 value. Figure 6 shows a typical G / G_0 versus e graph, where the curves are drawn for various soil type, i.e. for E_M/p_{LM} values varying between 5 (very loose sands) and 100 (rock). It is interesting to note that a) for 10 < E_M/p_{LM} <30 (usual soils), the curves are close to each other, b) that the well known Hardin and Drnevitch (1972) curve crosses the whole cluster.

It is also interesting to show the same graph in arithmetic scale for e (Figure 7), which is better understood by Civil Engineers. This graph obviously shows that the G values close to G_0 are only useful for Earthquake Engineering,

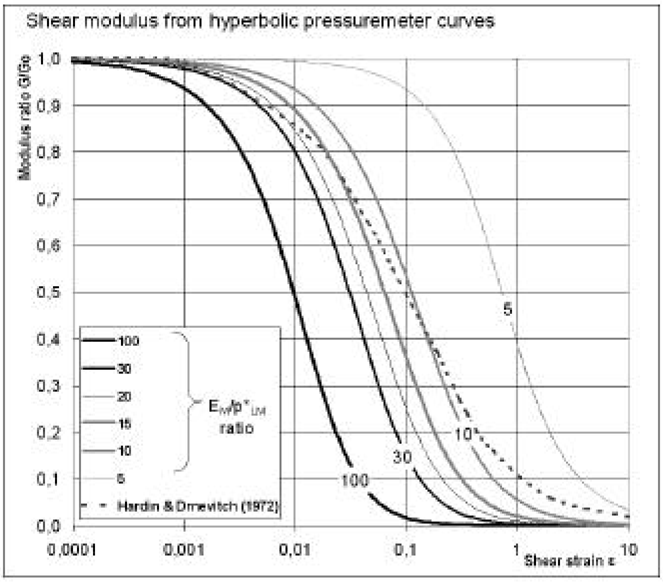

Figure 6. Typical G/ G_0 versus e graphs for various soils from the extrapolation of the pressuremeter hyperbolic law.

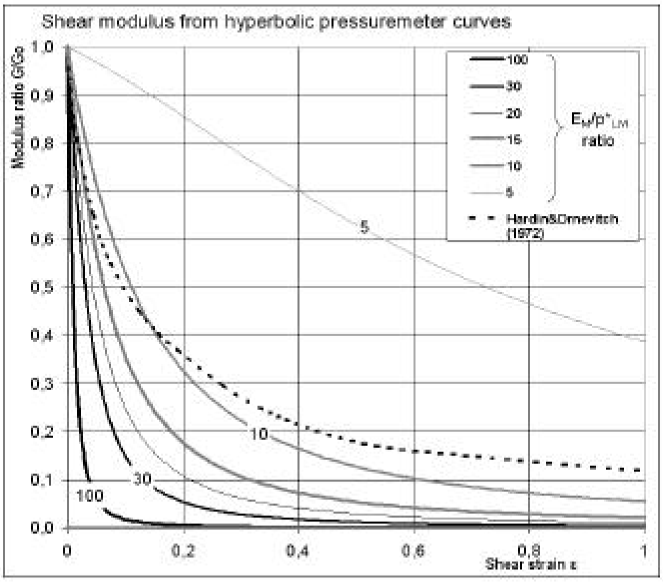

Figure 7. Graphs of G/ G_0 versus e in arithmetical scale.

4 CONCLUSIONS

Recent research work in the development of a “self-boring” methodology to carry out non-remoulded tests in most types of soils (Arsonnet & al., 2005) permits to get the most from the hyperbolic modelling of the MPM tests curves, especially when associated with a new sorting method to fix geotechnical categories (Baud, 2005) and a comprehensive use of the Xpressio® software. The authors propose this hyperbolic model as a more versatile constitutive law for a large range of soils, either granular or cohesive soils. Even a (G/G_0 ,ε) law can be established.

By extending Louis Ménard’s work, we hope that this new improvement combined with the automatic logging of the readings shall provide the geotechnical engineers with a more elaborated and cost/effective technique, so as to boost a new period of growth in the use of MPM testing.

5 BIBLIOGRAPHICAL REFERENCES

Apagéo (2006) *Xpressio program for MPM test interpretation*, User leaflet.

Arsonnet, G., Baud, J.-P. Gambin, M. (2005) Pressuremeter tests inside a self-bored slotted tube (STAF) (in French). In M. Gambin, J.-P. Magnan & P. Mestat (eds), *Proc. ISP5*, Vol.1, Paris, LCPC pp 31-45.

ASTM (2007), Test Method for Pressuremeter testing in Soils, D 4719, *Annual Book of ASTM Standard*, Vol. 04.08, West Conshohocken, PA, USA, ASTM.

Baguelin, F., Jézéquel, J., Shields, D. (1978) *The Pressuremeter and Foundation Engineering*, Clausthal Germany, Trans Tech Publications.

Baud, J.-P. (2005) A [log(E_M/p^*_{LM}), log(p^*_{LM})] diagram for spectral analysis of MPM tests results (in French). In M. Gambin, J.-P. Magnan & P. Mestat (eds), *Proc. ISP5*, Vol.1, Paris, LCPC pp 167-174.

Baud, J.-P. & Gambin, M. (2005) Deriving a single hyperbolic response to cylindrical expansion in a lithologically homogeneous soil by compilation of MPM curves (in French). In M. Gambin, J.-P. Magnan & P. Mestat (eds), *Proc. ISP5*, Vol.1, Paris, LCPC, pp 175-186.

Baud, J.-P., Gambin, M., Uprichard, S. (1992) Modeling and automatic analysis of a Ménard pressuremeter test. *Géotechnique et informatique*, Paris, Presses des Ponts.

Cassan, M. (1978) *Les essais in situ en mécanique des sols*. Eyrolles, Paris.

Duncan J. M. & Chang C. Y., (1970) Non linear analysis of stress and strain in soils, *ASCE, Journ. Soil Mechanics and Foundations Division*, 1970, vol. 96, n° SM5, pp 1629-1653.

D’Hemricourt, J. (2005) Pressuremeter test interpretation: from reciprocal volume method to double hyperbolic method (in French). In M. Gambin, J.-P. Magnan & P. Mestat (eds), *Proc. ISP5* Vol.1, Paris, LCPC.

Gibson, R. E. & Anderson, W. F. (1961) In-Situ Measurement of Soil Properties with the Pressuremeter, *Civil Engineering*, May, London.

Hardin, B. O. & Drnevich V. P. (1972) Shear modulus and damping in soils: design equations and curves, *ASCE, Journ. Soil Mechanics and Foundations Division*, 1972, vol. 98, n° SM7, pp 667-692.

Kondner, R. L. (1963), Hyperbolic stress-stain response. Cohesive soils, *ASCE, Journ. Soil Mechanics and Foundations Division*, 1963, vol. 89, n° SM1, Proc. Paper 3429, pp 115-143.

Ménard, L. (1957) *An apparatus for measuring the strength of soils in place*. Thesis for the degree of Master of Science, University of Illinois.

Ménard, L. (1963) Calculation of the Bearing Capacity of Foundations Based on the Results of Pressuremeter Tests (in French – expanded English abstract -), in *Sols Soils* n° 5, pp 9-28, Paris.

Ménard, L. (1975) Interpretation of pressuremeter tests results, *Sols Soils* n° 26, p.12 & 14 (a translation of a much older French user’s memo; several up-dated reprints were distributed under the name “General Memorandum No. D-60”).

Techniques Louis Ménard (1980) *Memo on the calculation and the submission of MPM results* (in French) User’s memo D-46, Paris.

Van Wambeke, A. & D’Hemricourt, J. (1975) Courbes pressiométriques inverses : méthode d’interprétation de l’essai pressiométrique *Sols Soils* n° 28, Paris.

Van Wambeke, A. & D’Hemricourt, J. (1978) Evolution dans l’interprétation de l’essai pressiométrique, Proc. 3rd Int. Congr. IAEG, sect.IV, Vol.1, pp.91-98.